

# NEW DENOISING SCHEME FOR MAGNETIC RESONANCE SPECTROSCOPY SIGNALS

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**Abstract**—A new scheme for denoising magnetic resonance spectroscopy signals is presented. The scheme is based on projecting noisy signals on different time-frequency domains, consecutively, and performing noise filtering operations in these domains according to the noise level. The evaluation of this scheme was performed using extensive simulation of magnetic resonance spectroscopy signals with different noise levels. It was observed that this scheme gives superior results that compensate for the excess computational requirements, especially for very low signal to noise ratio signals.

**Keywords**—magnetic resonance spectroscopy, denoising, time-frequency transform

## I. INTRODUCTION

Magnetic resonance spectroscopy (MRS) is a powerful investigative tool in chemical and biomedical applications and research. Many MRS signals, however, are faced with several difficulties such as their very low signal to noise ratio (SNR) and their overlapped resonances with different transverse relaxation time ( $T_2$ ) values. Signal averaging is used to increase the SNR, but this often results in a huge increase in MRS machine time, which may not be acceptable in many situations, particularly for unstable biological compounds where long replications are not feasible. Conventional Fourier transform techniques improve the spectral but they require long and tedious work by the spectroscopist [1]. Several methods for automatic and non-interactive MRS signal enhancement have been published [2–4]. Also, for automatic processing of MRS data, various model-fitting techniques are used to estimate the model parameters [5]. This is difficult because the model is nonlinear and the noise is relatively high. Thus, the noise should be reduced as a preprocessing step before applying the model-fitting technique [2, 4].

It is currently popular to use linear transforms such as Fourier, wavelet, and time-frequency (TF) transforms for denoising signals. The choice of the transform relies on the ability of the specified transform to compact the original signal in a small number of coefficients. This depends on the characteristics of both the signal and the transform. Since MRS signals have time-dependant spectra, their Fourier transform spreads along the frequency axis and thus hinders the utilization of this transform to denoise MRS signals. Various TF transforms such as Wigner distribution [6], Gabor transform [4], Zak transform [7], and SLTF transform [8] show a better compaction capability.

Regardless the type of transform, the following threshold-

ing scheme is generally used: transform the signal into the new domain, retain only the coefficients whose magnitudes are above a certain threshold (the threshold value is determined according to the noise level), then, perform the synthesis transform on the retained coefficients to get the noise-reduced signal.

However, in addition to the original signal, the retained coefficients still have some noise which cannot be separated from the original signal in this domain. In this paper, it is proposed to reproject the retained coefficients into another domain. This domain is chosen such that in this domain, the MRS signal is still compact and the remaining noise (or part of it) can be separated from the signal. Thus, performing a thresholding technique in the new domain may give further enhancement to the original signal. The SLTF transform with different window widths was selected in this paper since it has been shown to give better results than other TF transforms in MRS signal enhancement [8]. The proposed scheme, however, is applicable to any other TF transform.

In Section II, the proposed scheme is illustrated. Some results and discussions on MRS signals are given in Section III. The summary of the SLTF transform is given in the Appendix.

## II. THE DENOISING SCHEME

Any MRS signal,  $x(k)$ , can be modeled as a sum of  $Q$  exponentially-damped complex exponents plus noise [5], i.e.,

$$x(k) = \sum_{q=1}^Q c_q \exp \left( -\frac{t_k}{T_{2q}} + j2\pi f_q t_k \right) + z(k) \quad (1)$$

where  $c_q = b_q e^{j\phi_q}$  and  $b_q$ ,  $f_q$ ,  $\phi_q$ , and  $T_{2q}$  are the amplitude, frequency, phase and relaxation time of the  $q^{th}$  component, respectively,  $t_k = \Delta_t k$  with  $\Delta_t$  being the sampling period, and  $0 \leq k \leq L$ . The noise,  $z(k)$ , is well approximated by a complex white Gaussian noise with zero mean and standard deviation  $\sigma$ .

Each MRS component,  $c_q \exp \left( -\frac{t_k}{T_{2q}} + j2\pi f_q t_k \right)$ , is represented in the SLTF domain (3) by a spike centered at  $f_q$  on the frequency axis and has exponential decay in the time direction [8]. The compactness of this spike in the SLTF domain, however, depends merely on the width of the transform analysis window compared to the  $T_{2q}$  parameter of the MRS component. To illustrate this, let:

$$\begin{aligned} x_1(k) &= 32e^{55^\circ} e^{-\frac{t_k}{3}} + j3200\pi t_k \\ x_2(k) &= 32e^{55^\circ} e^{-\frac{t_k}{60}} + j3200\pi t_k \end{aligned}$$

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be two 1024-point noise-free one component MRS signals that differ only in their  $T_2$  values. In Figures 1-a) & 1-c),  $x_1$  and  $x_2$  are plotted in the TF domain using SLTF (3) with  $N = 32$ . In Figures 1-b) & 1-d)  $x_1$  and  $x_2$  are replotted with  $N = 256$ . Clearly, the choice of  $N = 32$  gives more compact representation than  $N = 256$  for  $x_1$ . On the other hand, the choice of  $N = 256$  gives more compact representation for  $x_2$ .

The above example shows how the compactness of MRS components in the SLTF domain changes with the value of  $N$  with respect to  $T_2$ . MRS signals, however, are generally composed of different components with different  $T_2$  values. Therefore, one SLTF transform with a certain  $N$  may compact certain MRS components more than the others.

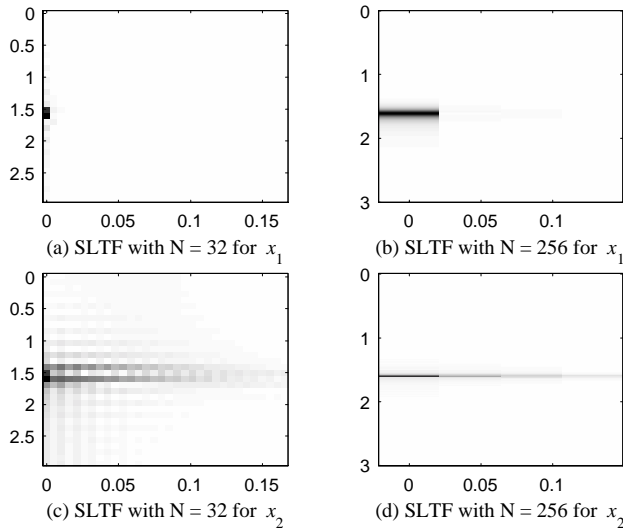


Fig. 1. noise-free one component MRS signal in TF domain

On the other hand, the MRS noise is white Gaussian and SLTF is a linear, near-orthogonal transform [9]. Thus, the MRS noise is fairly distributed among SLTF coefficients regardless of the choice of  $N$ . Typical MRS noise is plotted in Fig. 2 in the SLTF domain for  $N = 32$  & 256. Clearly, the noise is fairly distributed in both cases.

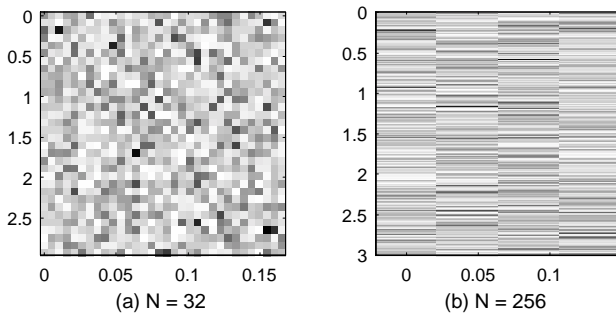


Fig. 2. Typical NMR noise in the SLTF domain

Thus, the SLTF domain for MRS signals will consist of two distinct areas: one containing noise only and the other containing the signal plus noise that has a higher amplitude than the former. Denoising the MRS signal is achieved by filtering out the former area. For example, if the two areas are

equal in space, approximately 50% of the noise is been cut off by filtering. The retained coefficients, however, still contain noise in addition to the original signal. Projecting the signal again in another SLTF domain with a different  $N$  will redistribute both the signal and the rest of the noise in such a way that some MRS components will be more compact while others will lose compactness. Thus, further signal enhancement can be achieved by filtering out the noise-only area in the new domain.

The above discussion suggests that to reduce the noise, the procedure is to:

1. estimate the noise variance  $\sigma$  using the last data points provided they are pure noise;
2. apply (3) with a certain  $N$  to obtain the SLTF transform of the MRS signal;
3. retain only the coefficients whose magnitudes are above a threshold value determined according to the SNR;
4. perform the SLTF synthesis transform (2) on the retained coefficients to obtain a noise-reduced sequence;
5. repeat steps 1-4 for another value of  $N$ .

In the next section, the proposed approach will be applied to simulated phosphorus MRS signal.

### III. RESULTS

Broad simulations were performed on synthesized representative MRS signals with different SNR values. The results obtained showed superior denoising performance for this scheme. As an example, the simulated phosphorus FID sequence (distorted to explore a wider range of the parameter values) [6, 7] which is widely used as test data, is used here. This sequence ( $S$ ) is 1024 points with a sampling period  $\Delta_t = 166.2\mu s$ .  $S$  is composed of six peaks. The frequencies, damping factors, amplitudes, and phases are given in Table I and the spectrum is shown in Fig. 3.

TABLE I  
PARAMETERS OF SIMULATED PHOSPHORUS FID

Peak	$f_q$ (Hz)	$d_q$ (ms)	$b_q$	$\phi_q$ ( $^\circ$ )
Reference	-1590	11	32,000	55
$P_i$	-600	2	10,000	83
PCr	-60	20	6,000	98.5
$\gamma$	240	6	9,000	107.5
$\alpha$	860	3	8,000	122.5
$\beta$	1900	5	4,000	153

A zero mean complex Gaussian noise,  $Z$ , is added to the sequence where  $\sigma = 3500$ . Since MRS signals are time varying, the SNR is defined as the signal energy over the energy of the noise in the observation period in decibels:

$$\text{SNR}_{\text{before}} = 10 \log (\|S\| / \|Z\|) = 5.0 \text{ db}$$

The corresponding spectrum, Fig. 4, shows that the noise totally corrupted the  $P_i$ ,  $\gamma$ ,  $\alpha$ , and  $\beta$  components.

To clean up  $S$  using the proposed scheme, three consecutive SLTF transforms with  $N = 16, 32, 64$  were used. The spectrum of the resulting noise-reduced sequence ( $S_1$ ) is shown in Fig. 5. The six peaks are now easily recognizable

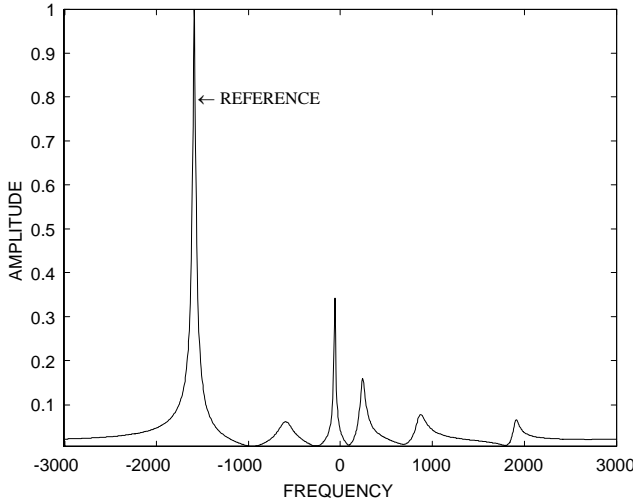


Fig. 3. Spectrum of the phosphorus FID without noise

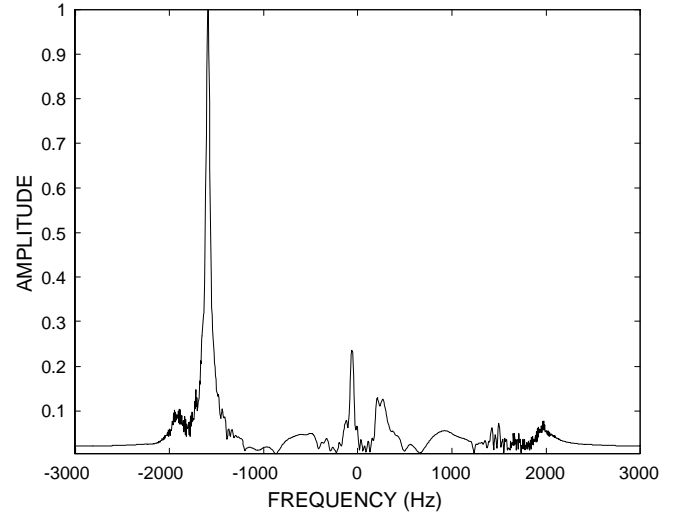


Fig. 5. Enhanced spectrum using the new scheme

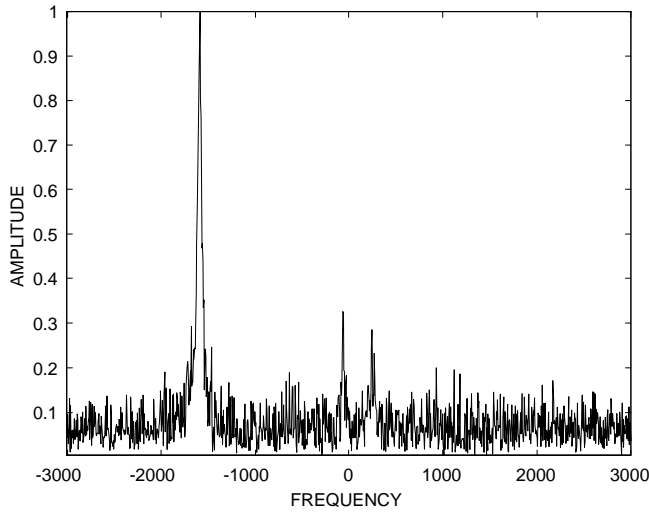


Fig. 4. Spectrum of the noisy phosphorus FID

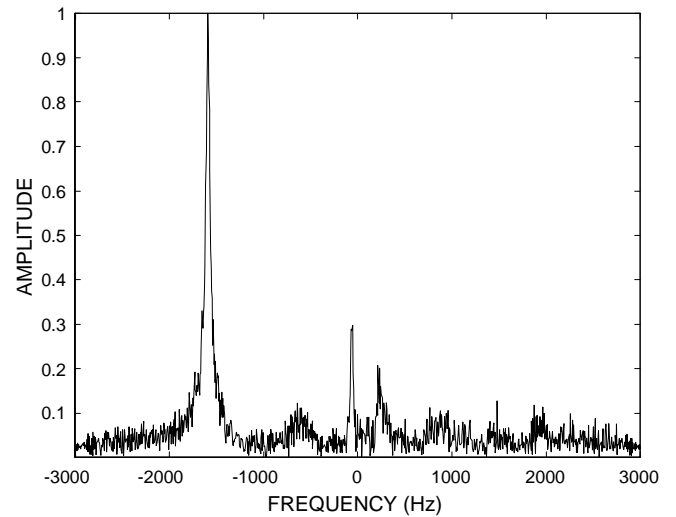


Fig. 6. Enhanced spectrum using SLTF  $N = 64$

in the spectrum of  $S_1$ . The SNR after processing is:

$$\text{SNR}_{\text{after}} = 10 \log (\|S\| / \|S_1 - S\|) = 15.35 \text{ db}$$

i.e., SNR gain of 10.35 db. For comparison, denoising the same sequence using SLTF with the same thresholding technique as in [8] with  $N = 16, 32, 64$  (each one alone) gave SNR = 10.19, 8.78, 10.53 respectively. The spectrum of the enhanced sequence using SLTF with  $N = 64$  (which is the best among the three) is shown in Fig. 6.

In another experiment, an 1024-point phosphorus FID sequence with diverse noise power was cleaned up by the proposed scheme using four consecutive SLTF transforms with  $N = 16, 32, 64, 128$ , respectively. The results are shown in Table II. The results of denoising the same sequence using a single SLTF with  $N = 16, 32, 64, 128$  is shown in the same table. Obviously, the proposed scheme gave far better results especially for low SNR (high  $\sigma$ ). For example, for  $\sigma = 3500$ , the SNR difference between the new scheme and the best SLTF with was 8.6 db.

#### IV. CONCLUSION

A new scheme has been introduced for denoising MRS signals. This scheme is based on projecting the noisy MRS signal on several TF domains, consecutively, and performing hard thresholding on all of them according to the noise power. The SLTF transform with different window widths was selected since it has been shown to give better results than other TF transforms in MRS signal enhancement. The proposed scheme was tested using simulated phosphorus FID signals distorted to explore a wider range of the parameter values. The proposed method produced superior results especially for low SNR at the expense of increasing the computation time. Thus, the proposed scheme may be useful in situations where SNR is low.

TABLE II  
 SNR ENHANCEMENT USING SLTF AND THE NEW SCHEME

$\sigma$	SNR <sub>before</sub>	SNR <sub>after</sub> using				
		SLTF with $N =$				New scheme
		16	32	64	128	
1500	9.54	11.9	13.1	12.1	12.7	19.0
2000	7.00	10.1	10.0	9.69	9.61	17.1
2500	4.97	8.96	8.6	9.97	7.08	16.3
3000	3.33	7.84	7.20	5.64	4.38	14.3
3500	2.00	4.73	4.54	4.07	3.66	13.3

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## V. APPENDIX: SLTF TRANSFORM

The SLTF is a stable, linear, critically sampled, time-frequency transform [9]. The discrete SLTF is defined for a finite extent ( $0 \leq k \leq L$ ) discrete signal  $x(k)$  as:

$$x(k) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{m,n} h(k - mN) \text{csin} \frac{\pi(k + \frac{1}{2})(n + \frac{1}{2})}{N} \quad (2)$$

for  $m = 0, \dots, M-1$  and  $n = 0, \dots, N-1$ .

where  $a_{m,n}$  are the transform coefficients,  $\text{csin}(\cdot)$  stands for  $\cos(\cdot)$  for even  $m$  and  $\sin(\cdot)$  for odd  $m$ .  $M$  and  $N$  are the number of analysis samples in time and frequency, respectively, where  $MN = L$ ,  $N$  controls the window-width, and  $h(k)$  is the discrete Gaussian function:

$$h(k) = \exp \left\{ -\frac{\pi}{2} \left( k - \frac{N-1}{2} \right)^2 \right\}$$

The analysis transform is given by:

$$a_{m,n} = \sum_{k=0}^{L-1} x(k) \gamma^*(k - mN) \text{csin} \frac{\pi(k + \frac{1}{2})(n + \frac{1}{2})}{N} \quad (3)$$

where  $\gamma(k)$  is the biorthogonal function to  $h(k)$ , i.e., it satisfies the biorthogonality condition:

$$\sum_{k=0}^{L-1} h(k - mN) \gamma(k) \text{csin} \frac{\pi(k + \frac{1}{2})(n + \frac{1}{2})}{N} = \delta_n \delta_m$$

Compared to other TF transforms, SLTF has several advantages regarding denoising MRS signals. First, SLTF is a linear TF transform that makes the synthesis transform quite easy to compute. This is in contrast to bilinear distributions (such as the Wigner distribution [6]) where difficulties are encountered in retrieving the signal from the TF domain. In addition, there is no cross-term interference such as obtained in the bilinear distributions.

Moreover, SLTF is a critically-sampled transform, which means that the transform coefficients are independent. Consequently, performing the synthesis transform after doing some filter operations like removing the noise is only a matter of matrix multiplication. This is in contrast to the over-sampled transforms where iterative methods are needed for the synthesis transform [10].

Compared to other critically-sampled TF transforms, like the Generalized Gabor Transform used in [4] to enhance MRS signals, SLTF has two major advantages: excellent localization of the biorthogonal function and good numerical stability (numerical stability is measured by the condition number which range from 1, optimal, to  $\infty$ . The condition number for the SLTF is always less than 1.2 for all values of  $M$  and  $N$ ) [8].

In addition, a fast algorithm to calculate the SLTF transform is presented in [11]. It requires only  $L \log_2 L - \frac{1}{2} \log_2 N$  multiplications and  $L(2 \log_2 L - \log_2 N - 3)$  additions for both the analysis and the synthesis transform computations.